

Gravity and Spacetime: Why do things fall?

A painless introduction to Einstein's theory of space, time and gravity

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Abstract

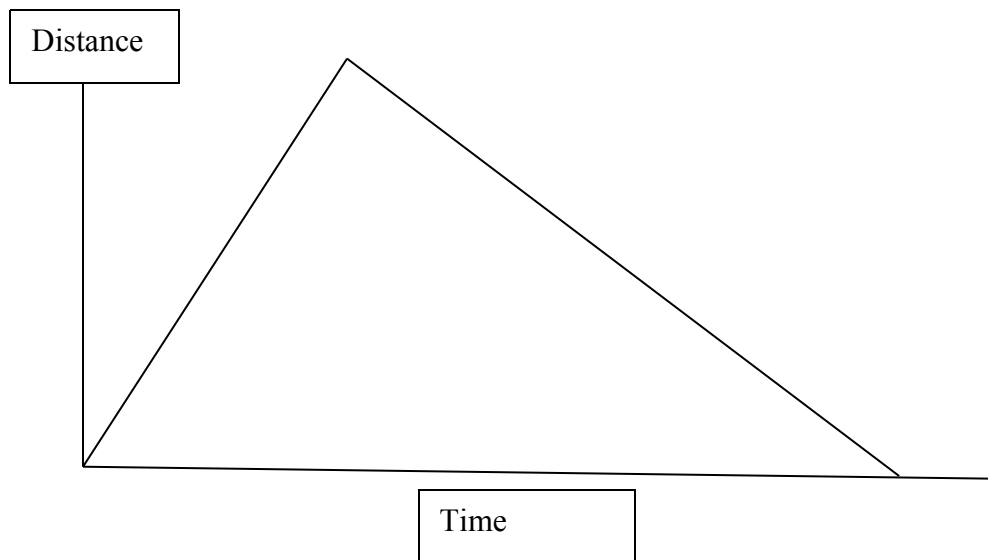
I present a simple description of Einstein's theory of gravity suitable for high school physics teaching. The concepts of spacetime, curved space and geodesics are described. Simple calculus is used to derive gravitational time dilation.

From Newton's Theory of Gravitation we imagine gravitational lines of force emanating from objects a bit like long rubber bands constantly pulling things together. Observations show that this view is incorrect. Sometimes it is a useful approximation, but fundamentally it is quite false.

Einstein's theory of gravity completely changes our view of gravitation. Gravitation is not a mysterious force that pulls objects together. Rather it is the force you have to apply to *prevent* things from taking a natural path. When you jump from the Leaning Tower of Gingin you feel nothing but increasing wind resistance!

Newton said that things move forever in straight lines at constant velocity unless some external force acts on them. Einstein said something quite similar and yet *totally different*. He said that freely falling objects always move in straight lines, and he said that space was curved! Sounds crazy? Well, it isn't crazy and it's not difficult to understand. But before we can understand what Einstein said, we need to introduce a few key ideas.

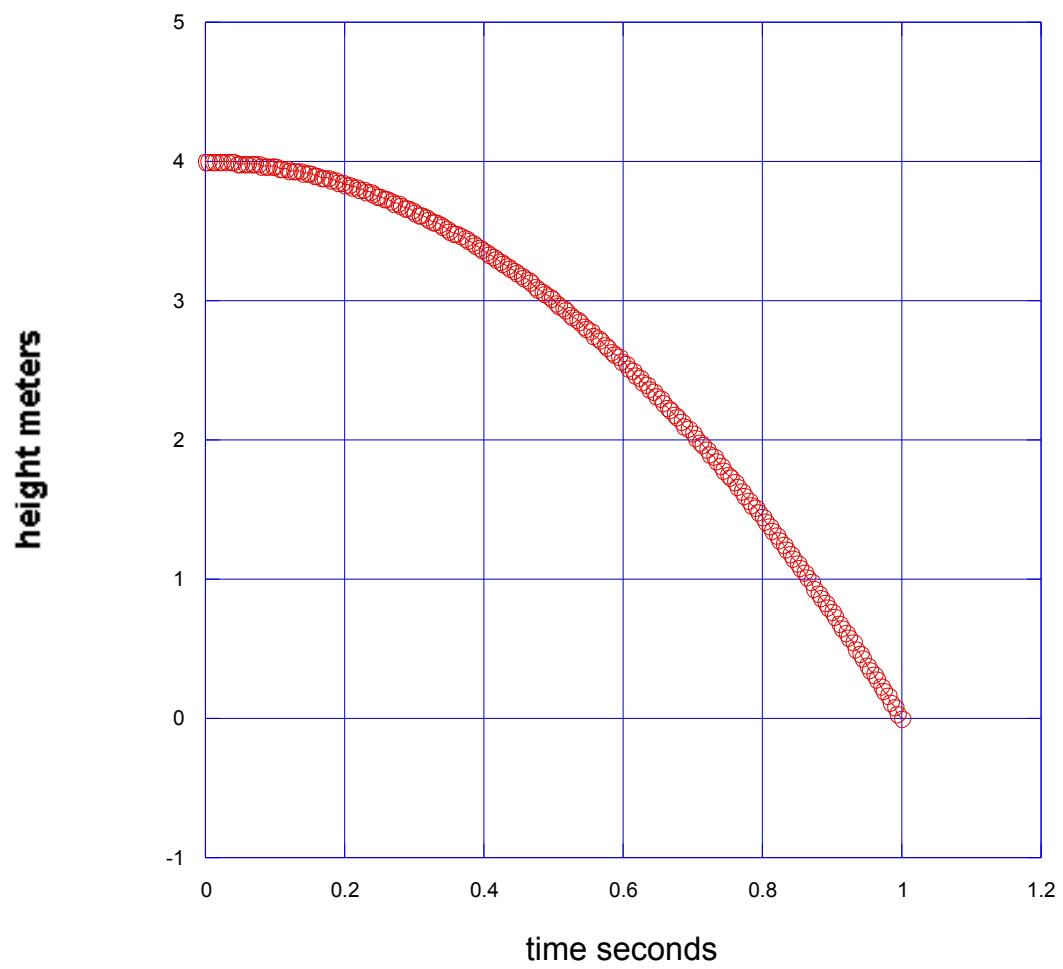
First Einstein said you must think of space and time together: this is called spacetime. Spacetime has three dimensions of space and one dimension of time. That means it is four dimensional. There are two problems with this. The first is that humans have a lot of trouble thinking about 4 dimensions. But we can make this easy if we decide to forget about two of the space dimensions and only think about say height or distance. Then we can draw spacetime in just two dimensions, one being distance, the other being time. We call a graph that shows motion in space a time a spacetime diagram and the path an object follows in spacetime is called a worldline. Soon we will look at the worldline of a falling object on a spacetime diagram. The diagram below shows the world line of a journey, first travelling fast (at constant speed) and then returning at a constant slower speed.

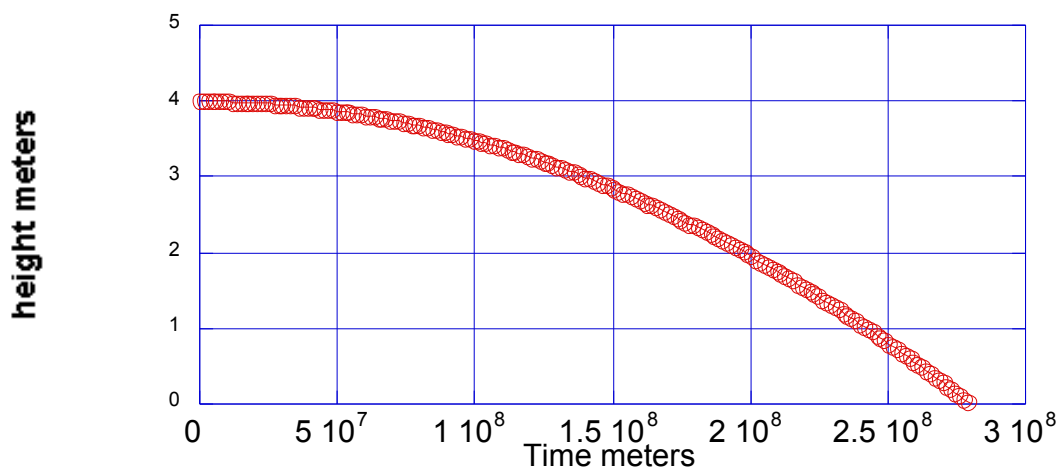


The second problem is that space and time are measured differently and so spacetime diagrams will look completely different if time is measured in seconds or in years.

But this problem is easily solved if we remember that speed connects space and time (like a speed of 100 kilometers per hour is a connection between kilometres and hours) If there was a special universal speed in the universe, it could be used to provide a universal link between space and time. Can you think of one?

The answer is of course, the speed of light. It is the absolute maximum speed anything can have and it is also a speed that is always the same whatever you do and wherever you measure it. Its value is very close to 300 million meters per second. If I use the speed of light to connect space and time, then I could plot a space time diagram using the units of say years for time and light years for distance, or seconds for time, and light seconds for distance, or the one I prefer, which is meters for distance and meters of light travel for time. In this case one second is 300 million meters.





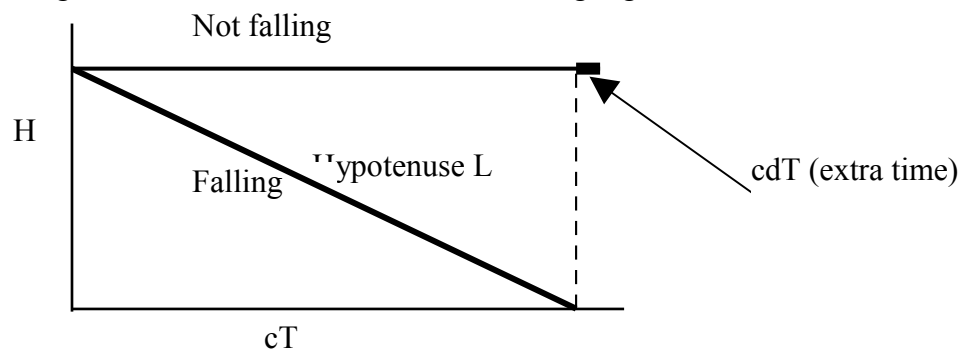
So now we have solved our problems about describing spacetime, but now we will jump to two new ideas: free fall and curved spacetime:

When you throw a stone it obviously does not go in a straight line. According to Newton, that is because gravity puts a force on it. Einstein says that's wrong! There is no force. Astronauts playing in the space station experience no force of gravity. If you jump on a trampoline you do not feel any force of gravity when you are in the air... you only feel it when you are landing. *Einstein says gravity is the force you have to apply to prevent free fall.* John Wheeler, the great physicist and colleague of Einstein who died in 2008, preferred to say *free float*, because usually freely moving objects like planets and stars do not fall but rather float endlessly through space. Einstein said that the paths of freely floating objects in space are straight lines in *spacetime*. They only deviated from their straight paths if a force like a rocket engine or the floor acts to stop them. How could that be? The path of the stone you throw is obviously curved, just like that of a planet! The answer is that what is curved is spacetime itself.

So what do we mean by curved space. Don't even try to think about 4 dimensions... that is better left to algebra. Stick with 2 dimensions so as not to strain your brain. A piece of paper is flat, the surface of a balloon is curved. Geometry is different on curved surfaces. Draw triangles on a balloon and see that their angles always add up to more than 180 degrees. Look at a straight line on a balloon. One way to make a straight line is to use a matchbox toy car to roll out a straight path, or use a piece of cotton. Straight lines are always the shortest path, just like airline routes on earth. Technically straight lines are called *geodesics*.

Now I have the final challenge: to explain why free fall is the shortest path in spacetime...why it is shorter than not falling at all.

We can deduce this by plotting a spacetime diagram of a falling stone, and ask how we would have to change the diagram to make the falling path a bit shorter than the not falling at all path. There is an easy way. If time runs slower for a stone on a shelf (when it cannot fall freely) then the worldline of the non-falling stone could easily be longer than the path of the falling stone. The spacetime diagram is so elongated...a few meters high and a few hundred million meters long, that the curved path of free fall could easily be shorter if there was just a tiny elongation of the non-falling path. While it takes sophisticated mathematics to calculate this precisely, it is easy to do a rough estimate of the size of this effect using high school calculus.



First we approximate the curved spacetime trajectory of the falling stone as the hypotenuse of a right angle triangle as shown above. The freely falling worldline is the hypotenuse and has length given by $L^2 = H^2 + c^2T^2$. Suppose that the non falling path is extended by a little bit dT . We ask how big dT has to be so that the nonfree fall path is at least equal to the falling path. Its length is $cT + cdT$. The minimum allowed value of dT must be such that $H^2 + c^2T^2 = (cT + cdT)^2$. Expanding this and remembering that dT is tiny so dT^2 is negligible, we have $H^2 + c^2T^2 = c^2T^2 + 2c^2TdT$ or $2c^2TdT = H^2$.

To link time to the velocity of freefall and gravity I want to use a formula that all physics teachers love : the speed of an accelerating object is given by $v^2 = 2as$, where a is acceleration and s is distance. For free fall on earth $a = g$, so $v^2 = 2gH$. Now we will use the approximation that v in this formula is the average free fall velocity, given by $v = H/T$. I divide the equation $2c^2TdT = H^2$ by T^2 on both sides and replace H^2/T^2 with v^2 . This gives $2c^2dT/T = v^2$. But $v^2 = 2gH$, so $2c^2dT/T = 2gH$ or $dT = gH/c^2$. This is a tiny change in time: it works out to be about 1 part in 10^{16} per meter. This tiny change in time is all it takes to make the free fall spacetime trajectory shorter than the non-free fall trajectory.

Using atomic clocks on rockets, scientists have found that the change in time with height is exactly this amount. We have been lucky in this calculation that the approximations averaged out and the change in time with height is exactly correct.

Einstein's theory says "*matter tells space how to curve, space tells matter how to move*". Wherever you have mass in the universe, spacetime is naturally curved. The curvature stretches beyond the mass. Spacetime is like a rubber membrane stretched by matter. When you stop things from following their natural trajectory, then you change both their position in space and their place in time. You have to apply forces

to change natural trajectories.

We can put this another way. The stone falls because free fall is the fastest way for the stone to become an old stone. Your head ages a bit faster than your feet because your feet are supporting you against slightly more curved spacetime closer to the earth's surface. You could stand on your head for half your life to compensate! Astronauts world lines are shortest because they are in free fall: they get to their destination (old age!) a few milliseconds before you.

The effect of gravity on time is called *gravitational time dilation*. It is an important effect for GPS navigation which uses clocks in spacecraft to work out your position on earth. Without correcting for curved spacetime the navigation system would not work.

Curved spacetime has many other effects. Mercury's orbital pattern is distorted by the curved space near the sun. This happens because the perimeter of the orbit is not given by the formula $\text{perimeter} = 2\pi D$. This formula is wrong because it is only true if space is flat. (test it with circles on a balloon!). Space is curved everywhere, so this formula is always wrong, even though it is a useful approximation. The effect of curved space on orbits is to make an elliptical orbital pattern rotate: in the solar system this is called *precession of the perihelion*. Another effect of curved space is called *gravitational lensing*. Light like anything in free fall follows geodesics, and when light passes through curved space it also takes the shortest path. Images of distant galaxies are distorted by gravitational lensing. By measuring the lensing distortions astronomers use this effect to map out matter in the universe. It provides an especially powerful tool for mapping the dark matter that envelopes all galaxies.

The understanding of Einsteinian physics is central to our understanding of everything around us. We need to abandon the obsolete Newtonian concepts while using the tools of Newtonian physics (like $F=ma$ and $s=ut+\frac{1}{2}at^2$) as convenient approximations.