Time and Periods

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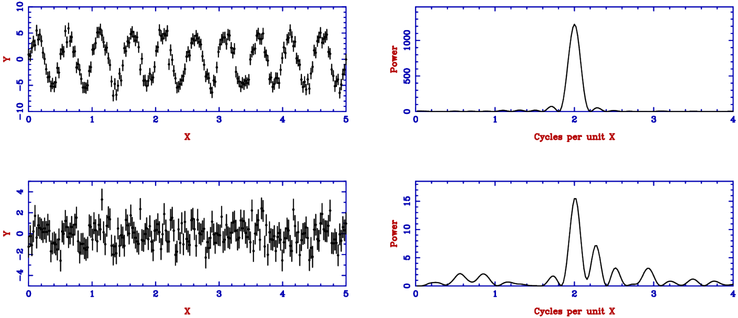

Outline

Time is important in the study of binary stars.

It is an area of near-infinite confusion.

In this lecture:

1. Time systems
2. Ephemerides, "$O - C$" diagrams
3. Period determination, aliasing.
4. Periodograms

Time systems

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UTC "Coordinated Universal Time". Always an integral number of seconds behind TAI, kept to within 0.9 seconds of UT1. Leap second (delays) are added irregularly, at midnight on either December 31 or June 30. (Most recent: June 30 2012). UTC replaces the older GMT. Today $\mathrm{TAI} - \mathrm{UTC} = 35\,\mathrm{s}$.

Astronomical time – I

JD "Julian Date" ["Julian" as in Julius Caesar]. UTC expressed in days since noon on January 1, 4713 BC (Julian calendar; Nov 24, 4714 BC Gregorian).
Midnight 10/11 Sep 2012 $\rightarrow \mathrm{JD} = 2456182.5$.

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$$\mathrm{MJD} = JD - 2400000.5.$$

MJD $= 0$ at 0h, Nov 17, 1858. MJD has a computational benefit in terms of precision because of roundoff error. e.g. to write a JD to $1\,\mu\mathrm{s}$ precision requires 18 digits.

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Confusion potential: 8/10. Potential penalty: 0.5 days
In tables in papers it is quite common to quote JDs with the leading "24" dropped off. Normally this is specified, but if not, is it a JD or an MJD?

Astronomical time – II

UTC is discontinuous, but can easily be corrected to TAI. However TAI is no use on its own for astronomical times because Earth's motion around the Sun leads to variable arrival times (range: $\pm 500\,\mathrm{s}$).

1. Correct to centre of Sun ("heliocentre") $\rightarrow$ HJD, "Heliocentric Julian Day" (also HMJD)

2. But the centre of the Sun moves irregularly too because of the planets (range $\pm 2\,\mathrm{s}$):
   - $\rightarrow$ correct to centre of mass of solar system ("barycentre") $\rightarrow$ BJD, BMJD.

3. However, TAI corrected to the barycentre of the solar system is still not what we want ...

Astronomical time – III

Earth's eccentric orbit and perturbation from the other planets means that its velocity varies and that it moves through a varying gravitational potential. TAI therefore is in the language of GR a proper time whereas we want a coordinate time. This led to:

TT "Terrestrial Time", replacing earlier "ET" and "TDT". TT is directly related to TAI:

$$\mathrm{TT} = \mathrm{TAI} + 32.184.$$

TDB "Barycentric Dynamical Time". This is finally the time we are after. It is TT corrected for GR effects (range $\pm 0.002\,\mathrm{s}$), making it suitable for astronomical time measurements.

TCB "Barycentric Coordinate Time". Essentially identical to TDB but advances at a slightly higher rate. TDB is much more commonly used.

Time recipe

1. Establish the mid-time of your image or spectrum, typically in UTC. Make sure you know whether JD or MJD is being used.

2. Try to find out if it is actually accurate; computer clocks can be terrible . . . some can drift over one minute a day!

3. Work out whether you want HJD or BJD. HJDs are usually computed from JD(UTC). If you want BJD, you need JD(TDB). Although BJD(TDB) should be the final time to aim for, HJD(UTC) is often needed for backwards compatibility.

4. Apply the appropriate light-travel time correction. In very precise work you should worry about the precision of coordinates, $\delta\theta$:

$$\delta t \leq 2.4 \times 10^{-3} \left(\frac{\delta\theta}{1''}\right) \mathrm{s}.$$

Time pitfalls

I have seen all of the following:

1. MJD / JD error. Cost: 0.5 days, spotted, luckily.
2. Failure to correct for light-travel time. Cost: missed eclipse of target on v.expensive satellite.
3. Using UTC rather than TDB to derive barycentric times. Cost: $\sim 60\,\mathrm{s}$ leading to invalid published claim of relative X-ray/optical phasing.
4. Imprecise coordinates in pulsar work. Cost: spurious claim in Nature of planet with 6 month period.
5. Many instances of plain poor times (10 minutes out in one case.) Cost: spurious results polluting the literature.

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If your research involves precision timing, take care!

Time resources

1. USNO: the US Naval Observatory has good information on time systems.

2. SOFA: "Standards Of Fundamental Astronomy". Provides a good library of subroutines for coordinate and time transformations. (Python fans: see pysofa)

3. For super-accurate work (i.e. pulsars), and for testing simpler routines, look up the state-of-the-art package, TEMPO2.

Ephemerides – I

We specify the location of a binary in its orbit with an ephemeris as follows (for the star NN Ser):

$$\mathrm{BMJD(TDB)} = 47344.0254693(7) + 0.13008012180(2)E,$$

where within parentheses are the 1σ uncertainties in the last digit and E is the "cycle number".

The precise meaning of E varies:

1. often (as here) it marks the time of the deepest (primary) eclipse,
2. but it is also quite often the time of maximum recession velocity of the brighter component.

Ephemerides – II

If the period changes we need a quadratic ephemeris:

$$T = T_0 + P_0 E + C E^2.$$

The instantaneous period at cycle E is

$$P = \frac{dT}{dE} = P_0 + 2CE.$$

The rate of change of period is therefore

$$\dot{P} = \frac{dP}{dT} = \frac{dE}{dT}\frac{dP}{dE} = \frac{2C}{P},$$

so

$$C = \frac{1}{2} P \dot{P}.$$

Ephemeris uncertainties

$$T = T_0 + P_0 E + C E^2.$$

Let the covariance between two quantities A and B be denoted $V(A, B)$, then the uncertainty in the predicted time of cycle E, $\sigma_T = V(T, T)^{1/2}$, is given by:

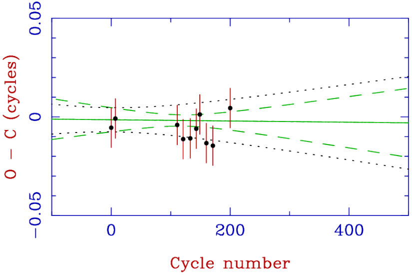
$$\begin{aligned}\sigma_T^2 &= V(T_0, T_0) + V(P_0, P_0)E^2 + V(C, C)E^4 + \\ &\quad 2V(T_0, P_0)E + 2V(T_0, C)E^2 + 2V(P_0, C)E^3,\end{aligned}$$

but people only rarely quote the cross-coefficients.

If creating a linear ephemeris, you can choose the zero point of E to minimise $V(T_0, P_0)$ to avoid the issue.

Top:

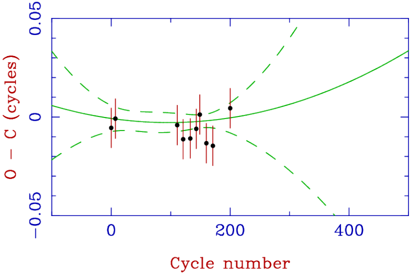
Linear ephemeris, dashed lines show $\pm 3\sigma$ on prediction. Dotted line: result if cross-coefficient is ignored.

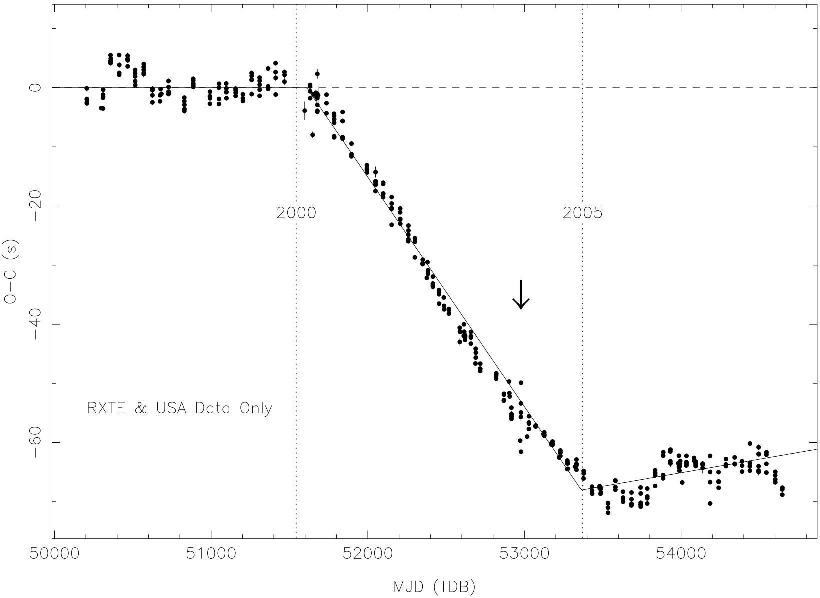


Bottom:

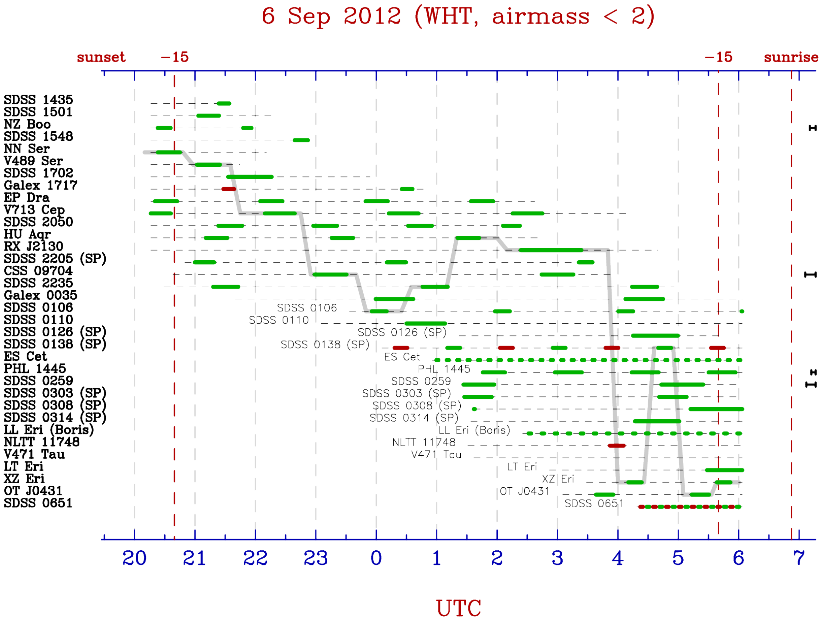
Quadratic ephemeris fit to the same data. Statistical uncertainty "blows up" outside span of data.

Favour linear ephemerides unless you have good reason not to.





Wolff et al (2009), $O - C$ of X-ray binary, EXO 0748-676


6 Sep 2012 (WHT, airmass < 2)
sunset
−15
−15
sunrise
SDSS 1435
SDSS 1501
NZ Boo
SDSS 1548
NN Ser
V489 Ser
SDSS 1702
Galex 1717
EP Dra
V713 Cep
SDSS 2050
HU Aqr
RX J2130
SDSS 2205 (SP)
CSS 09704
SDSS 2235
Galex 0035
SDSS 0106
SDSS 0110
SDSS 0126 (SP)
SDSS 0138 (SP)
ES Cet
PHL 1445
SDSS 0259
SDSS 0303 (SP)
SDSS 0308 (SP)
SDSS 0314 (SP)
LL Eri (Boris)
NLTT 11748
V471 Tau
LT Eri
XZ Eri
OT J0431
SDSS 0651
20
21
22
23
0
1
2
3
4
5
6
7
UTC

Period determination: aliasing

Suppose one had the following measurements of eclipse times, the last taken a year after the first two:

E	Time (BMJD)
0	551200.345±0.002
4	551200.834±0.002
X	551569.347±0.002

What is P and cycle number X?

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First two measurements give

$$P = \frac{0.834(2) - 0.345(2)}{4} = 0.1222 \pm 0.0007.$$

Therefore:

$$X = \frac{569.347(2) - 200.345(2)}{0.1222(7)} = 3019 \pm 17.$$

Period determination: aliasing

If $X = 3019$:

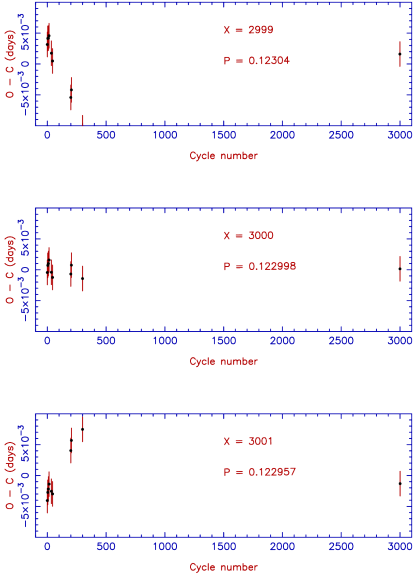
$$P_{3019} = \frac{569.347(2) - 200.345(2)}{3019} = 0.1222265(2).$$

but given the ± 17, we could equally well have:

$$\begin{aligned} P_{3018} &= 0.1222671(2), \\ P_{3020} &= 0.1221861(2), \\ P_{3021} &= 0.1221456(2), \\ &\ldots \end{aligned}$$

These are period aliases. They crop up time and again in one form or other.

Breaking aliases



Reliable selection of unique periods requires avoidance of cycle count ambiguity.

Typically this requires enough measurements at one epoch to jump the gaps to others.

Periodograms

For a set of radial velocity measurements (t_i, v_i, σ_i for $i = 1, 2, \ldots N$) of a circular orbit one fits the model

$$v_i^* = \gamma + A \sin(2\pi f\, t_i) + B \cos(2\pi f\, t_i),$$

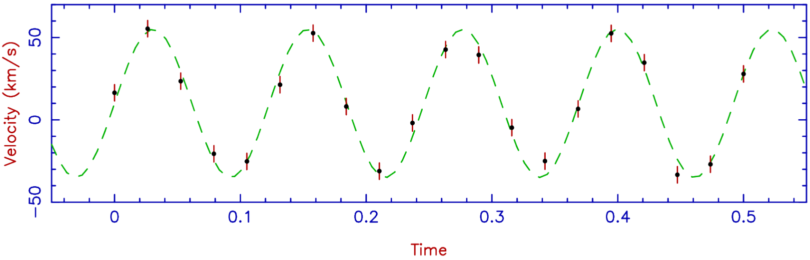
and computes $\chi^2_{\min[\gamma,A,B]}(f)$. This is $\leq \chi^2_{\min[\gamma]}$ for a constant model. Thus

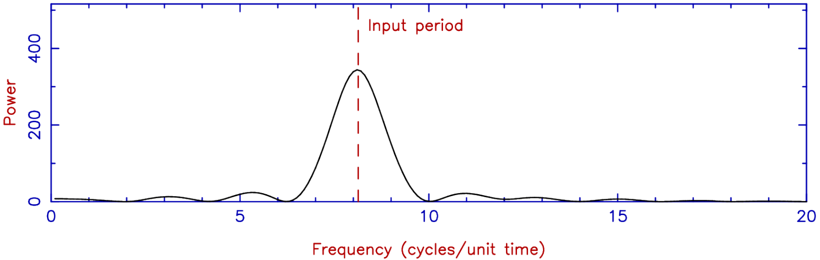
$$S(f) = \frac{\chi^2_{\min[\gamma]} - \chi^2_{\min[\gamma,A,B]}(f)}{2},$$

the improvement in χ^2 of a constant+sinusoid vs a constant-only model, is a statistic indicative of favoured frequencies/periods.

Apart from the optimisation with respect to γ, this is the "Lomb-Scargle periodogram" (hence the factor 2).

Periodograms





Window functions

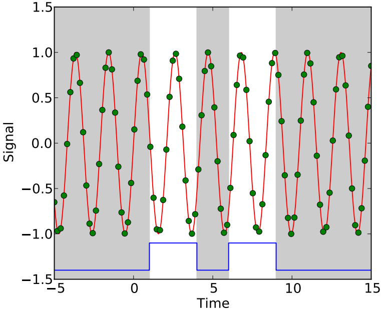
Can view a dataset O as an infinite ideal sequence S viewed through a "window" W set by observing constraints so that

$$O = WS.$$

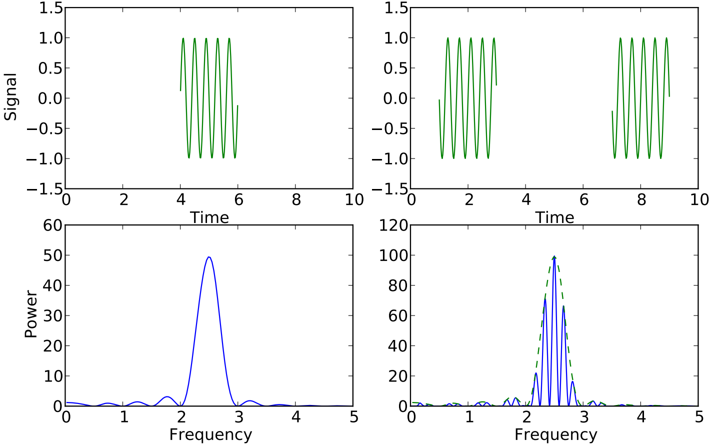
By the convolution theorem

$$\tilde{O} = \tilde{W} * \tilde{S},$$

where $\sim$ denotes a Fourier transform (FT). Thus the observed FT $\tilde{O}$ is the convolution of the ideal one $\tilde{S}$ with the "window function" $\tilde{W}$, set by your sampling.



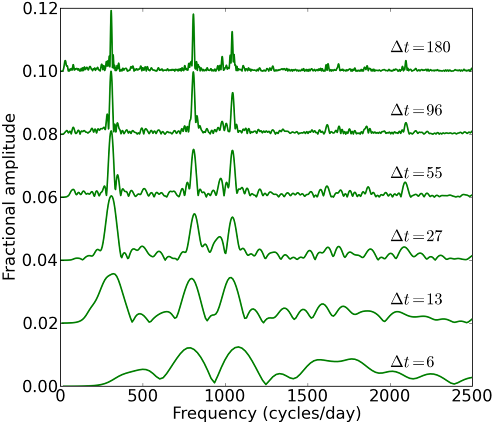
Window function examples



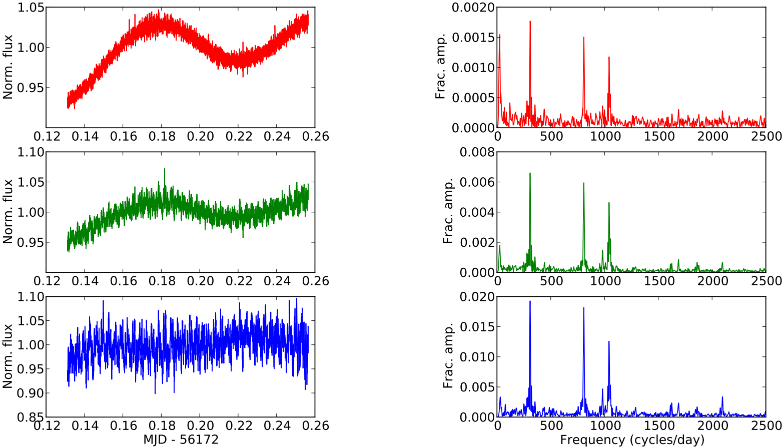
Periodograms: resolution vs time

The frequency resolution of a periodogram varies inversely with the duration of observations (cf uncertainty principle).

To resolve two peaks separated by Δf, one needs a timebase $\Delta t > 1/\Delta f$.

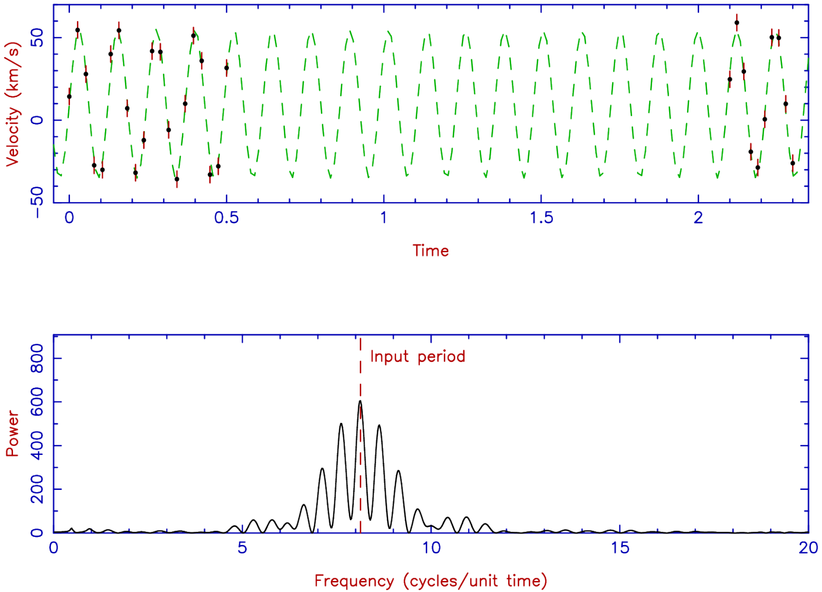


Periodograms can simplify . . .



WD/dM binary: ellipsoidal M dwarf, pulsating white dwarf.

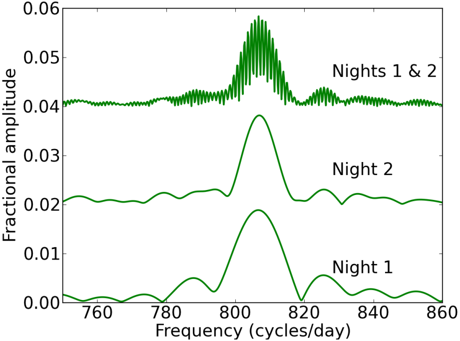
Periodograms & aliasing



Periodograms & aliasing

In a simple case as at the right (white dwarf pulsator) the aliases are spaced by 1 over the time difference between the two sets of data.

In this case the duration per night is not long enough to establish a unique alias from one night to the next.



Periodogram tips

The L-S periodogram is just one possibility; non-sinusoidal signals may require more complex models (e.g. box-car models for planet transits).

Deciding whether a signal is real and which alias is correct can be tricky – see papers by Alex Schwarzenberg-Czerny.

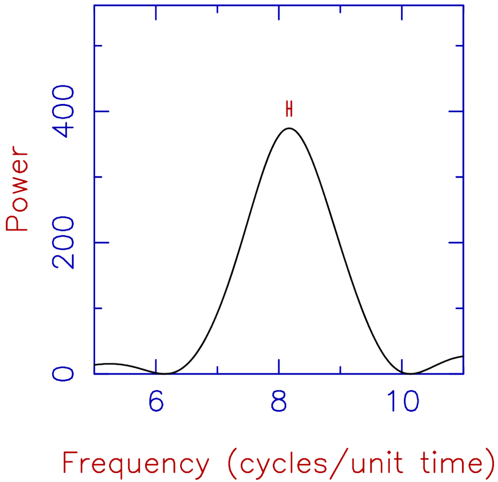
Statistical results are mostly-based upon singly periodic signals; for treatment of multiply periodic signals, e.g. "pre-whitening", see papers on asteroseismology.

Beware red noise, aka $1/f$ noise or flickering. See papers on X-ray binaries and AGN and Vaughan, 2010, MNRAS, 402, 307 for some cautionary tales.

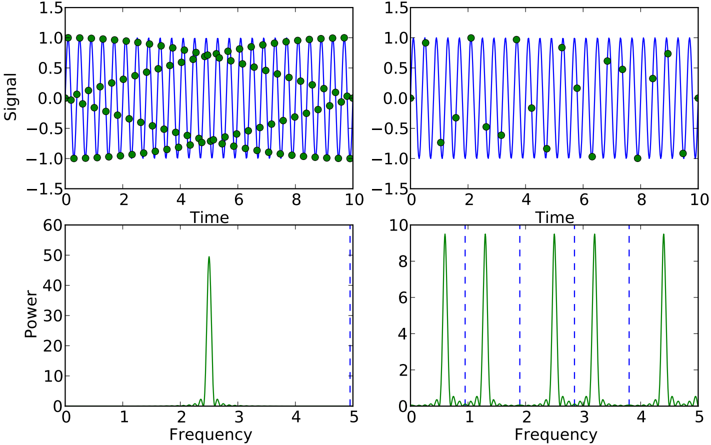
Period precision

Common misconception:
The width of an alias peak corresponds to the precision with which one can measure the frequency.

It is related, but only loosely. One can often do much better.



Aliasing from under-sampling



Conclusions

- Time and period analysis hold many unpleasant surprises for the unwary. You have been warned!
- Familiar connections between time and frequency come up in period measurement.
- A thorough understanding of these concepts is important for optimising observations of binary stars.